Lightweight Construction of S-Boxes

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8 nov. 2016

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Encryption

Send a secret message...

Alice

My secret Diary

Her secret Diary

Bob

Encryption

But mind the enemy!

Alice

My secret Diary

Her secret Diary

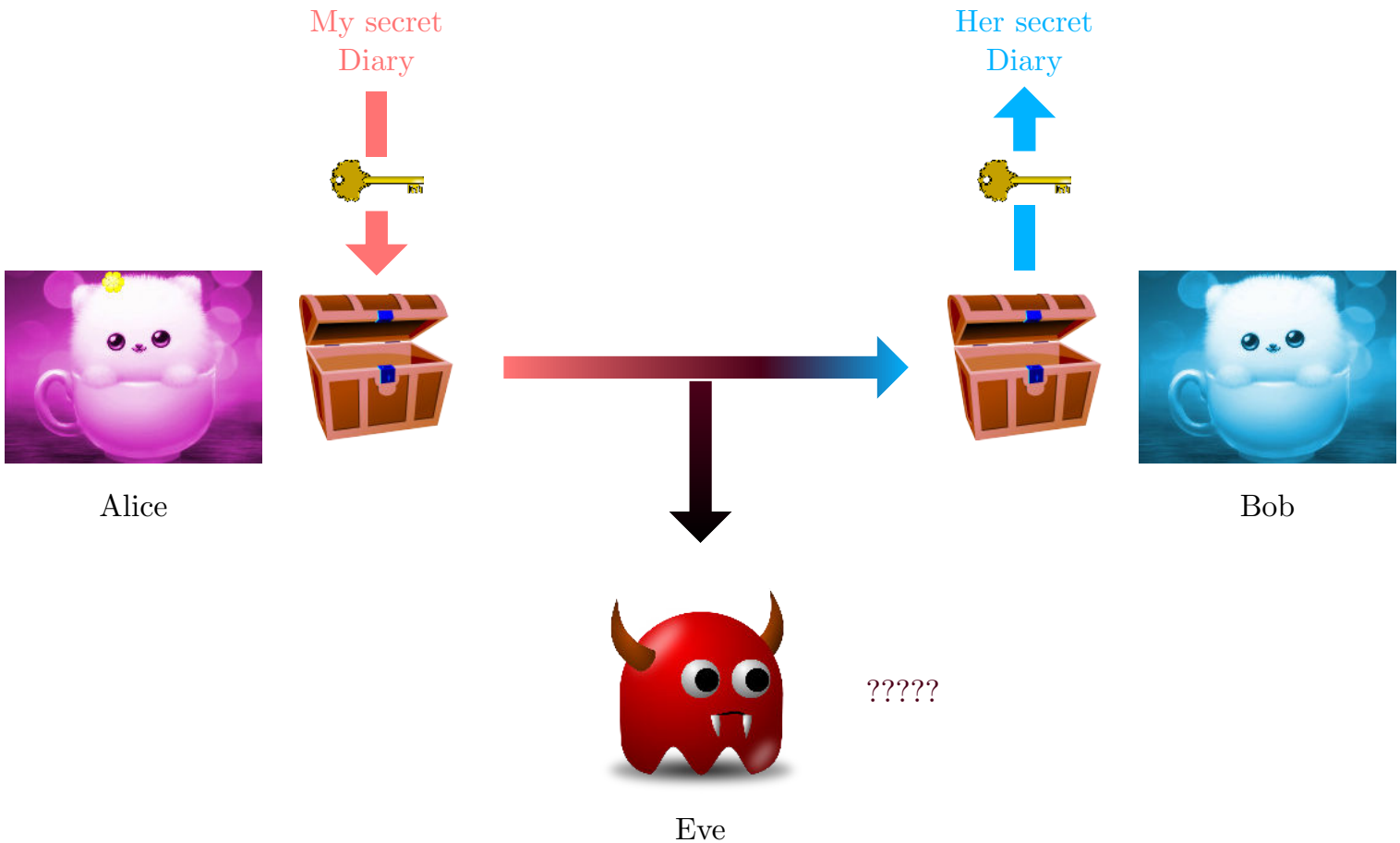
Bob

Hmmm, interesting...

Eve

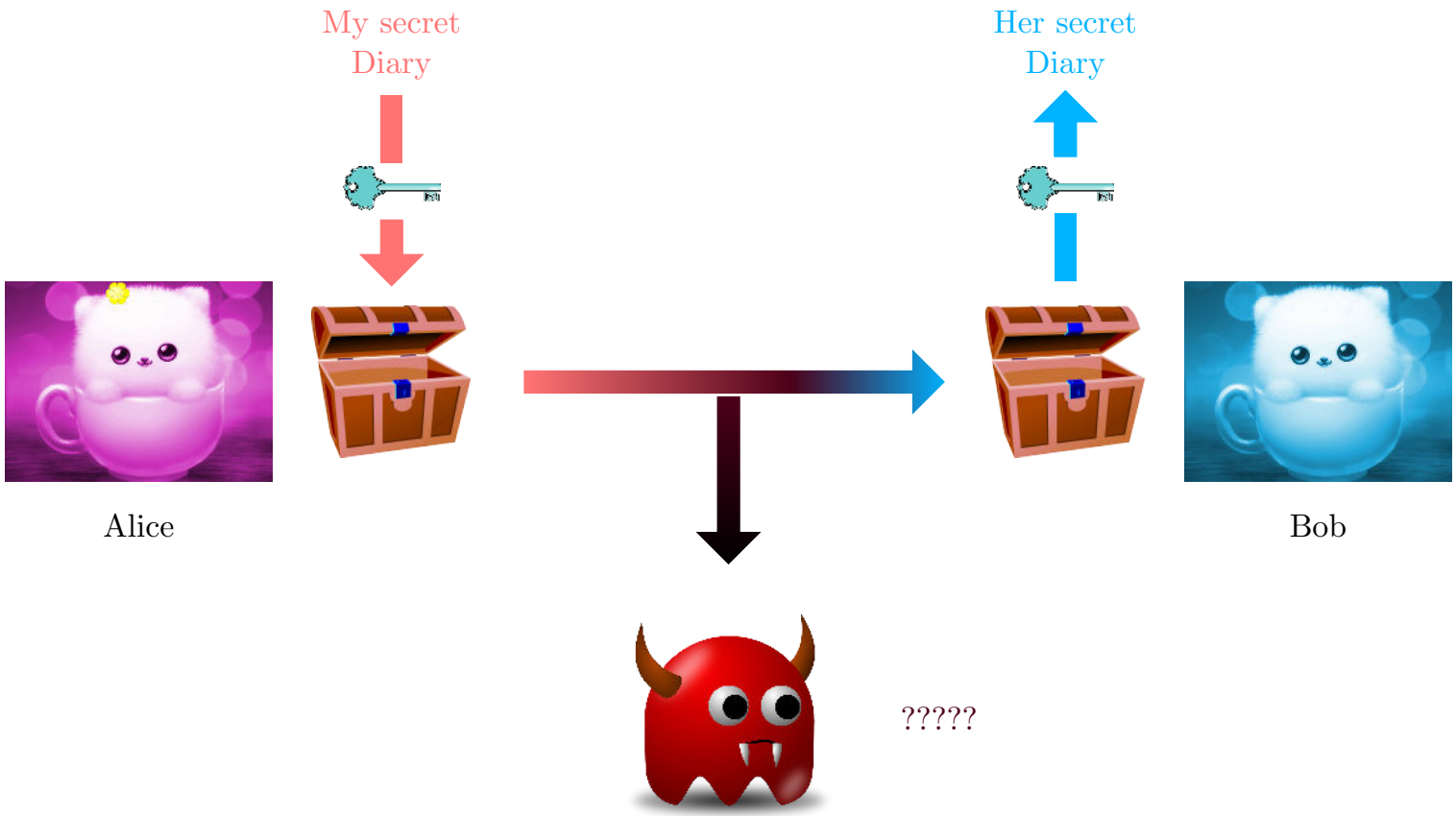
Encryption

Use encryption



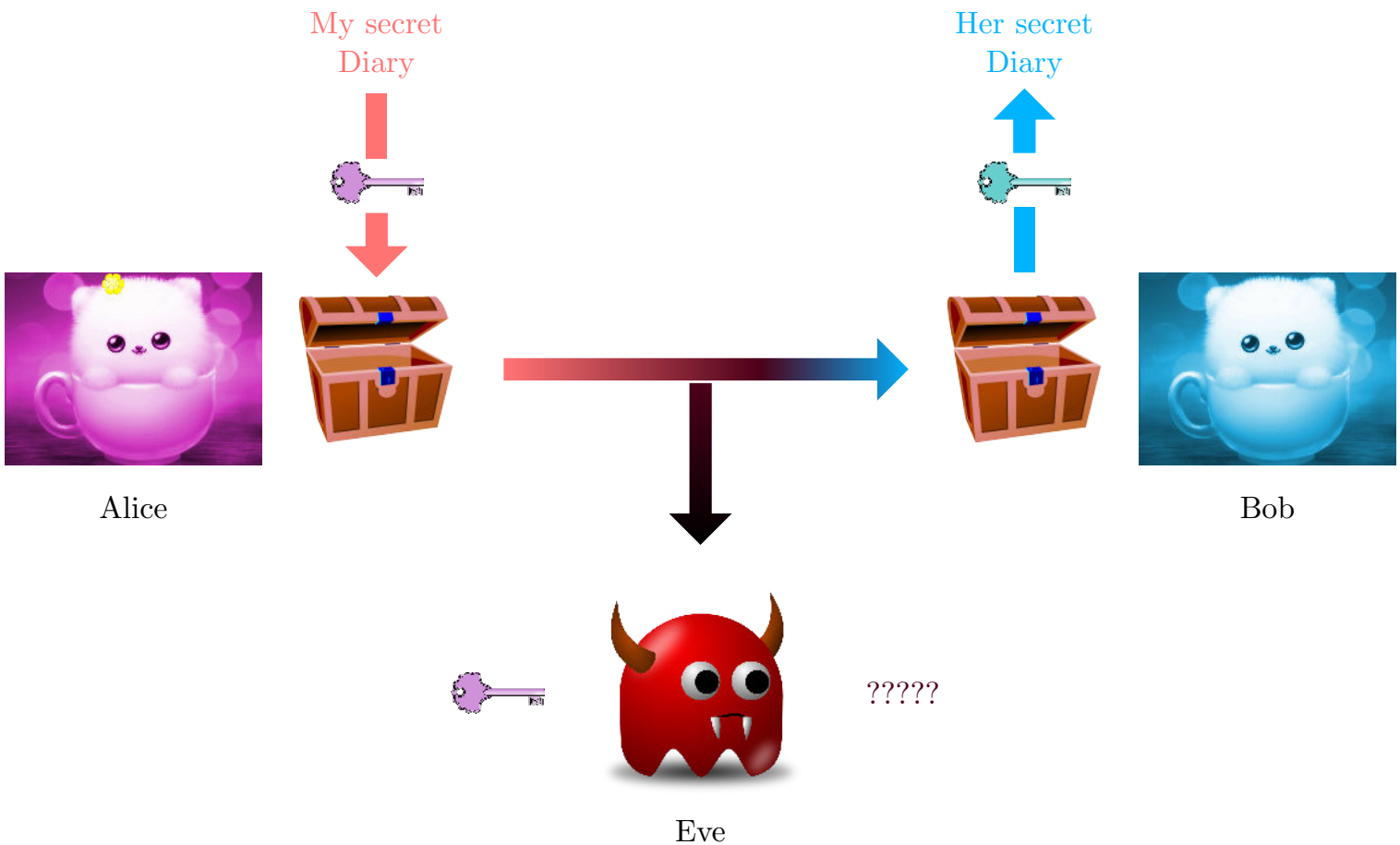
Encryption

Private key encryption



Encryption

Public key encryption

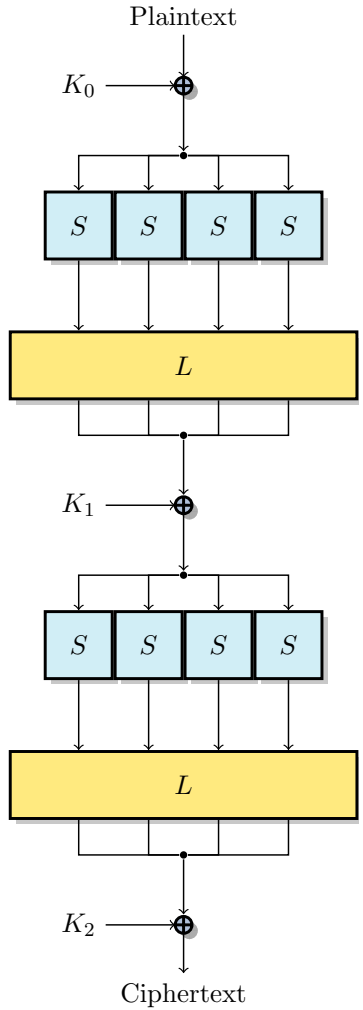


Symmetric Encryption: Security Criteria

Shannon's Criteria

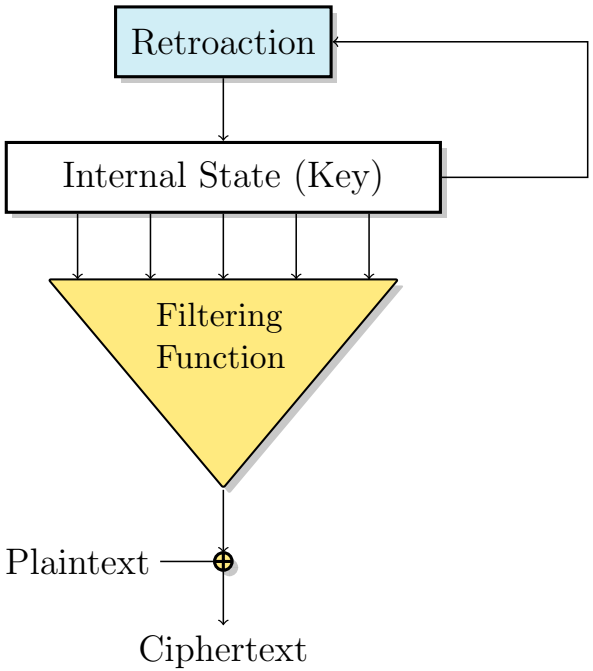
$$p \in \mathbb{F}_2^n \Rightarrow c \in \mathbb{F}_2^n$$

1. Diffusion
   - $\forall i, j$, p_i affects c_j.
   - Can be achieved using linear functions.
2. Confusion
   - Relation between p and c must be complex.
   - Requires non-linear functions.
   - Implemented as tables: S-Boxes.

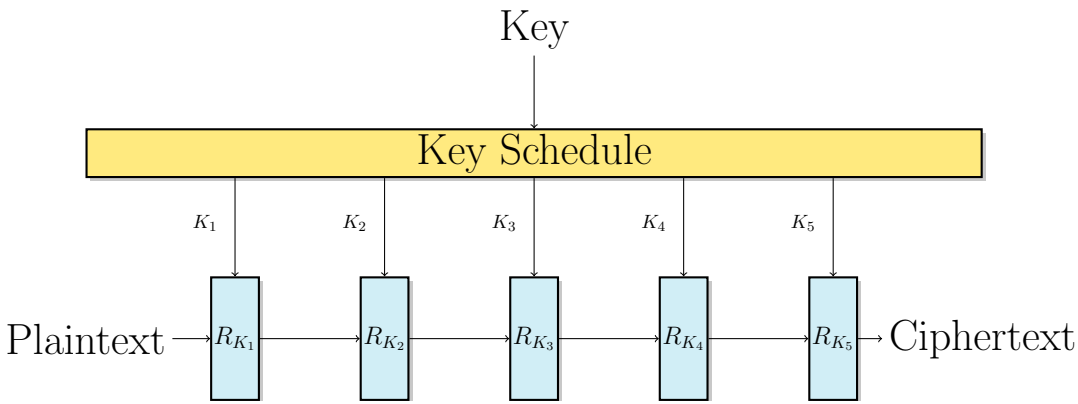


SPN Encryption

Stream & Block Ciphers

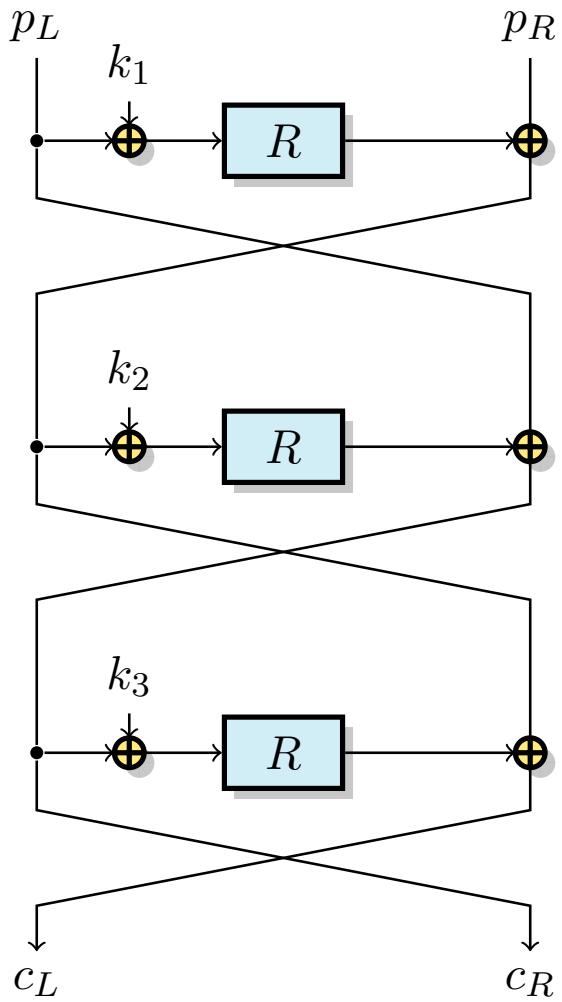


Stream Cipher



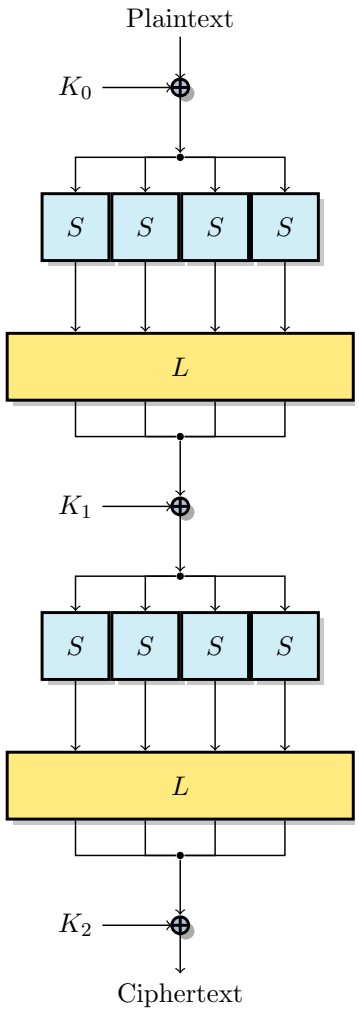
Block Cipher

Feistel Ciphers



- Lucifer/DES (H. Feistel, IBM, 1974)
- R built recursively
- Involution up to key ordering

SPN Ciphers



- Rijndael/AES (J. Daemen, V. Rijmen, 1988)
- Succession of confusion/diffusion layers
- Good for parallelism and easy to implement

Proven Security?

No NP-Complete Problem

- No reduction to an NP-complete problem
- No proven security
- Hypothesis: distinguishing from a random permutation is hard

Hard to Formalise

- Formal definition:
  - Chosen Plaintext Attack.
  - Cipher indistinguishable from a PRP $\Rightarrow$ secure against CPA.
  - i.e.: No Turing machine gives a different answer if given the cipher or a PRP.
- In practice:
  - How to define a "random" permutation ?
  - New property of random permutations $\Rightarrow$ new attack
  - We need cryptanalysis

Statistical Attacks

- Distinguish from random $\Rightarrow$ attack
- Lots of properties:
  - Differential attacks
  - Linear attacks
  - Algebraic attacks
  - Subset attacks
  - ...
- Most efficient: differential and linear
- Very similar

Differential Attacks

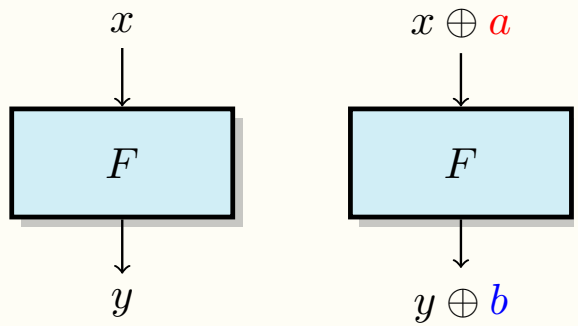
Definition: Differential Uniformity

Let F be a function over $\mathbb{F}_2^n$. The table of differences of F is:

$$\delta_F(a \to b) = \#\{x \in \mathbb{F}_2^n | F(x \oplus a) = F(x) \oplus b\}.$$

Moreover, the differential uniformity of F is

$$\delta(F) = \max_{a \neq 0, b} \delta_F(a \to b).$$



We will also consider:

$$\delta_{\min}(F) = \min_{a \neq 0} \max_{b} \delta_F(a \to b).$$

- F is resistant against differential attacks if $\delta(F)$ is small
- $\delta_F(a \to b)$ is even
- $\delta(F) = 2$ for APN functions

Table of Differences

a\b	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	2	0	0	0	0	2	2	0	4	0	4	2	0	0
2	0	0	2	4	2	0	0	0	2	4	0	0	0	0	2	0
3	0	2	2	0	4	0	0	0	2	0	0	2	0	0	4	0
4	0	2	0	0	0	0	0	2	0	4	0	2	4	2	0	0
5	0	4	0	0	2	0	0	2	0	0	0	4	0	2	2	0
6	0	0	0	4	0	4	0	0	0	4	4	0	0	0	0	0
7	0	0	2	0	0	4	0	2	2	4	0	0	0	2	0	0
8	0	2	2	2	0	2	0	0	2	0	0	0	2	0	2	2
9	0	0	2	2	2	0	2	0	0	0	0	0	0	2	2	4
10	0	4	2	0	0	0	4	2	0	0	2	0	0	0	0	2
11	0	0	0	0	0	2	2	0	0	0	2	2	2	4	2	0
12	0	0	2	2	2	2	0	0	2	0	0	2	2	0	0	2
13	0	0	0	2	2	0	2	2	2	0	0	0	0	0	2	4
14	0	0	0	0	0	0	4	0	2	0	2	4	0	2	0	2
15	0	2	0	0	2	2	2	4	0	0	2	0	2	0	0	0

All values are even:

$$S(x) \oplus S(x \oplus a) = b \iff S((x \oplus a) \oplus a) \oplus S(x \oplus a) = b$$

Linear Attacks

Definition: Linearity

Let F be a function over $\mathbb{F}_2^n$. The table of linear biases of F is:

$$\lambda_F(a, b) = \sum_{x \in \mathbb{F}_2^n} (-1)^{a \cdot x \oplus b \cdot F(x)}.$$

Moreover, the linearity of F is

$$\mathcal{L}(F) = \max_{a,b \neq 0} |\lambda_F(a, b)|.$$

- F is resistant to linear attacks if $\mathcal{L}(S)$ is small

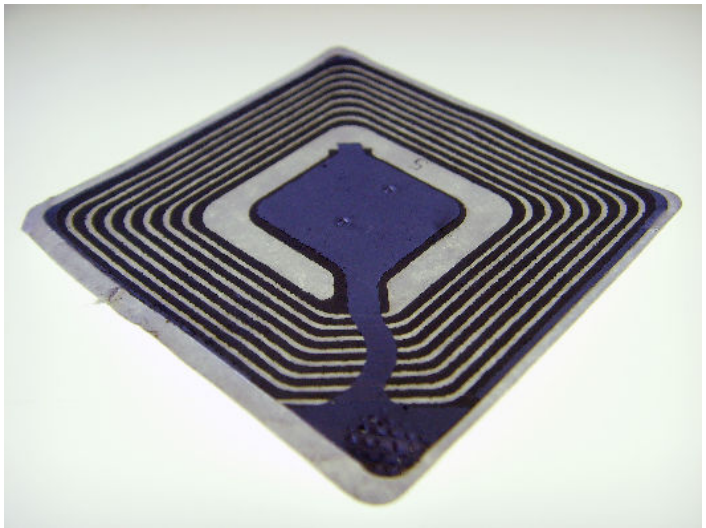
What Now?

We have good ciphers, considered secure and well studied with a powerful background theory: What now ?

- Still a lot of theory
- Cryptanalysis: Find new attacks
- ...
- Fit constrained specifications:
  - FHE
  - Side-channel attacks
  - Lightweight
  - ...

Lightweight Cryptography

- Secure and fast ciphers
- But too costly for dedicated environments...
- Useful for connected devices



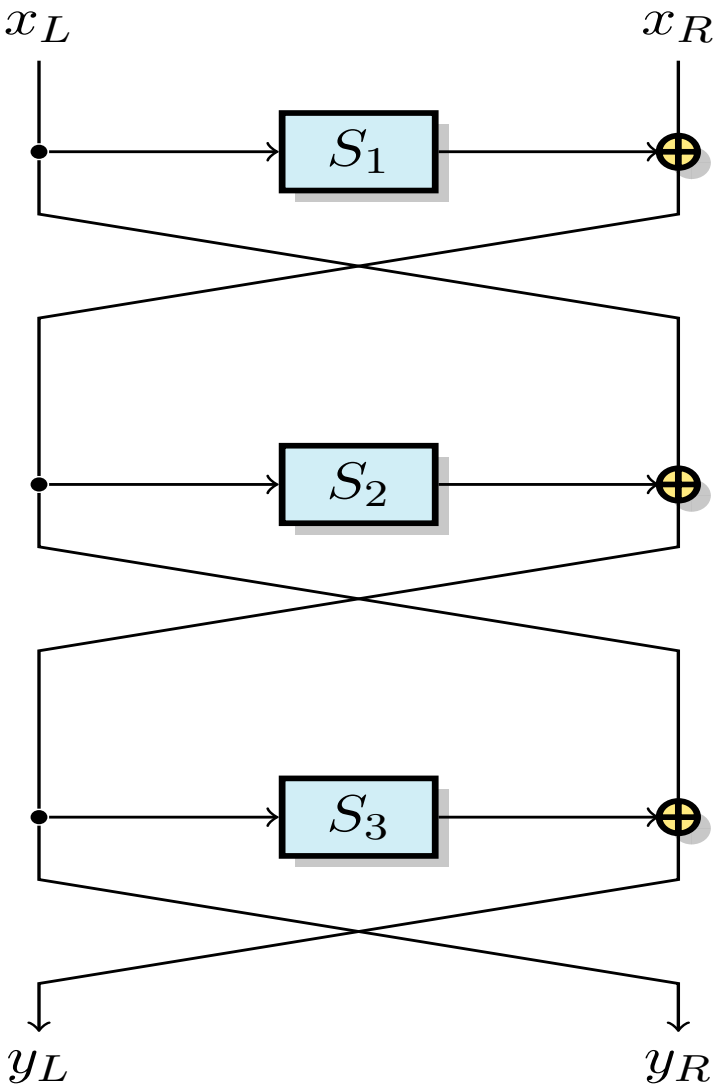
- Size of an RFID chip: $< 10\,000$ GE
- Smallest implementation of AES: $\sim 10\,000$ GE

Directions for Building S-Boxes

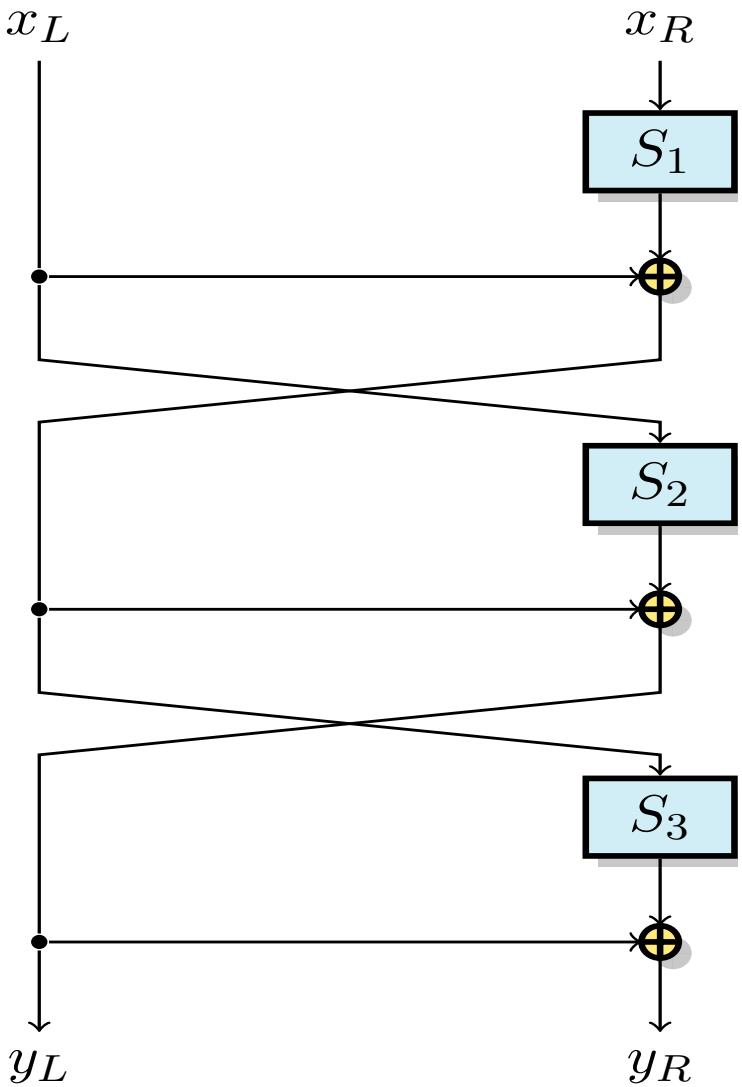
Problem: S-Box implementations are expansive

- Standard S-Box size: 8 bits (operations on bytes)
  - Implementation remains costly
- Smaller S-Boxes for a lesser cost:
  - Software implementation (table): Smaller table
  - Hardware implementation: Less logic gates
- But requires more rounds for same security
- Can we find a trade-off ?

Building Bigger S-Boxes From Small Ones



Feistel



MISTY

Objective of this Work

- Construction of S-Boxes using Feistel and MISTY networks
  - Construction of 8-bit S-Boxes from 4-bit ones
  - Trade-off between implementation cost and security

Results

- Determine the best properties reachable using MISTY and Feistel
  - Applied to 8-bit S-Boxes
- From theory to practice: Contruction of lightweight S-Boxes

Feistel and MISTY to Build Ciphers

- Initially used to define block ciphers (keyed networks)
- Well studied, many known results:

$$\mathrm{MEDP}(F_K) = \max_{a \neq 0, b} \frac{1}{2^k} \sum_{K \in \mathbb{F}_2^k} \frac{\delta_{F_K}(a \to b)}{2^n}$$

$$\mathrm{MELP}(F_K) = \max_{a,b \neq 0} \frac{1}{2^k} \sum_{K \in \mathbb{F}_2^k} \left(\frac{\lambda_{F_K}(a,b)}{2^n} \right)^2$$

- For MISTY and Feistel:

$$\mathrm{MEDP}(S_i) \leq p \Rightarrow \mathrm{MEDP}(F) \leq p^2$$

$$\mathrm{MELP}(S_i) \leq q \Rightarrow \mathrm{MELP}(F) \leq q^2$$

- Caution! Doesn't work when the key is fixed !

Feistel and MISTY with Fixed Key: Limits of MEDP

Example

- 3-round MISTY network.
- $S_1 = S_2 = S_3 = [A, 7, 9, 6, 0, 1, 5, B, 3, E, 8, 2, C, D, 4, F]$.
- $\delta(S_i) = 4$, $\mathrm{MEDP}(S_i) = 2^{-2}$.
- $\mathrm{MEDP}(F) \leq 2^{-4}$.
- For every key, there exists a differential with probability 2^{-3}.

- A bound on MEDP means:
  1. Choose an input and an output difference.
  2. For any chosen key, differential probability is low.
- No bound when the key is chosen before the differences!
- When building S-Boxes, there is no key, i.e. $K = 0$.

Feistel: Prior Results

Theorem (Li et Wang, CHES 2014)

Let F be a 3-round Feistel network with internal functions S_1, S_2 et S_3, then

- $\delta(F) \geq 2\delta(S_2)$
- $\delta(F) \geq 2^{n+1}$ if S_2 is not a permutation
- Pour $n = 4$, $\delta(F) \geq 8$, and if $\delta(F) = 8$, then $\mathcal{L}(F) \geq 64$
- $\delta(F) = 8$ and $\mathcal{L}(F) = 64$ is reachable

Feistel: New Results

Theorem

- $\delta(F) \geq \delta(S_2)\max(\delta_{min}(S_1), \delta_{min}(S_3))$
- $\delta(F) \geq 2^{n+1}$ if S_2 is not a permutation
- $\delta(F) \geq \max_{i\neq 2, j\neq i,2}(\delta(S_i)\delta_{min}(S_j), \delta(S_i)\delta_{min}(S_2^{-1}))$ if S_2 is a permutation with $\delta_{min}(S) = \min_{a\neq 0}\max_b \delta_S(a \to b)$
- This bounds depend on all 3 S-Boxes

Pour $n = 4$

- $\delta(F) \geq 8$, tight
- $\mathcal{L}(F) \geq 48$, $\mathcal{L}(F) \geq 64$ if $\delta(F) < 32$

MISTY: New Results

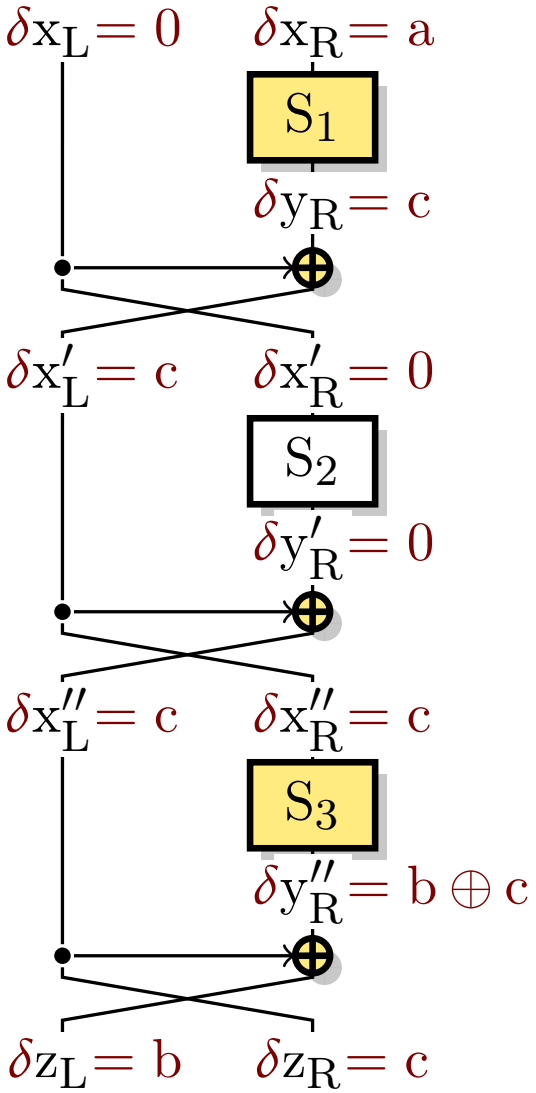
Theorem

- $\delta(F) \geq \delta(S_1) \max(\delta_{min}(S_2), \delta_{min}(S_3))$
- $\delta(F) \geq 2^{n+1}$ if S_1 is not a permutation
- $\delta(F) \geq \max_{i \neq 1, j \neq i,1}(\delta(S_i)\delta_{min}(S_j), \delta(S_i)\delta_{min}(S_1^{-1}))$
  if S_1 is a permutation
  with $\delta_{min}(S) = \min_{a \neq 0} \max_b \delta_S(a \to b)$
- There was no prior result for MISTY with fixed key

Pour $n = 4$

- $\delta(F) \geq 8$, tight
- $\mathcal{L}(F) \geq 48$, $\mathcal{L}(F) \geq 64$ if $\delta(F) < 32$

Sketch of Proof



Proposition

$$\delta_F(0 \,\|\, a \to b \,\|\, c) = \delta_{S_1}(a \to c) \times \delta_{S_3}(c \to b \oplus c)$$

Proof

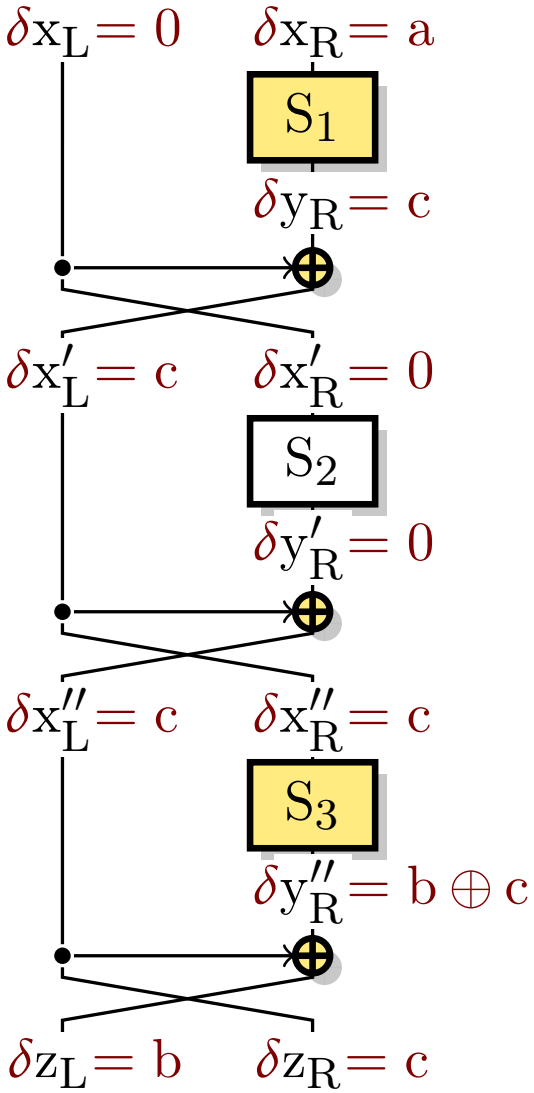
$$F(x_L \| x_R) \oplus F(x_L \| (x_R \oplus a)) = b \| c$$

$$\Leftrightarrow \begin{cases} S_3(S_1(x_R) \oplus x_L) \oplus S_3(S_1(x_R \oplus a) \oplus x_L) = b \oplus c, \\ S_2(x_L) \oplus S_1(x_R) \oplus x_L \oplus S_2(x_L) \oplus S_1(x_R \oplus a) \oplus x_L = c \end{cases}$$

$$\Leftrightarrow \begin{cases} S_3(S_1(x_R) \oplus x_L) \oplus S_3(S_1(x_R \oplus a) \oplus x_L) = b \oplus c, \\ S_1(x_R) \oplus S_1(x_R \oplus a) = c \end{cases}$$

$$\Leftrightarrow \begin{cases} x_R \in D_{S_1}(a \to c) \\ x_L \in S_1(x_R) \oplus D_{S_3}(c \to b \oplus c) \end{cases}$$

Sketch of Proof



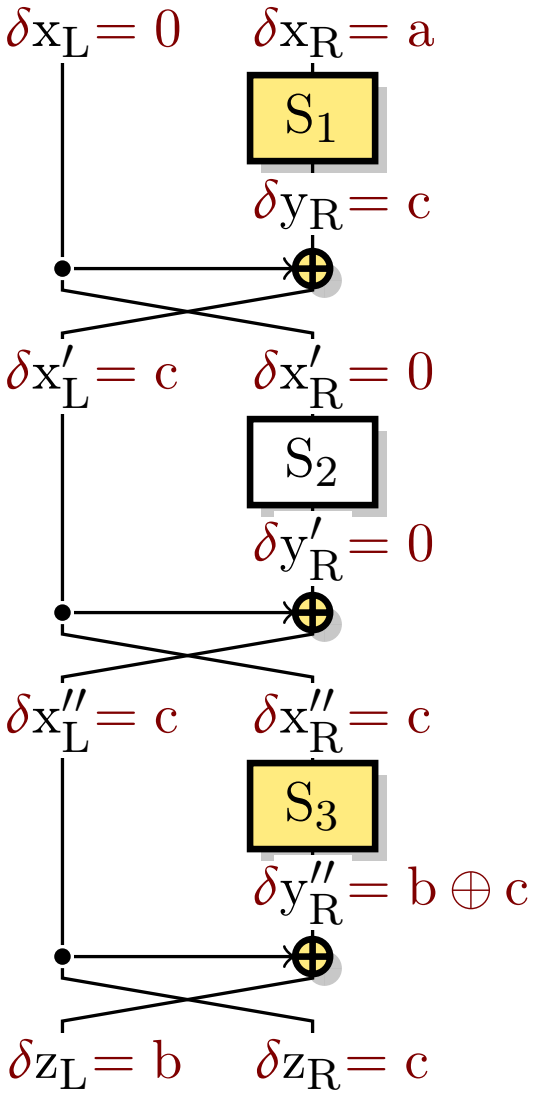
Proposition

$$\delta_F(0 \,\|\, a \to b \,\|\, c) = \delta_{S_1}(a \to c) \times \delta_{S_3}(c \to b \oplus c)$$

Application: if S_1 is not bijective

- Fix $b = c = 0$, $\delta_{S_3}(0 \to 0) = 2^n$
- Choose a such that $\delta_{S_1}(a \to 0) \geq 2$
- $\delta(F) \geq \delta_F(0 \,\|\, a \to 0 \,\|\, 0) \geq 2^{n+1}$

Sketch of Proof



Proposition

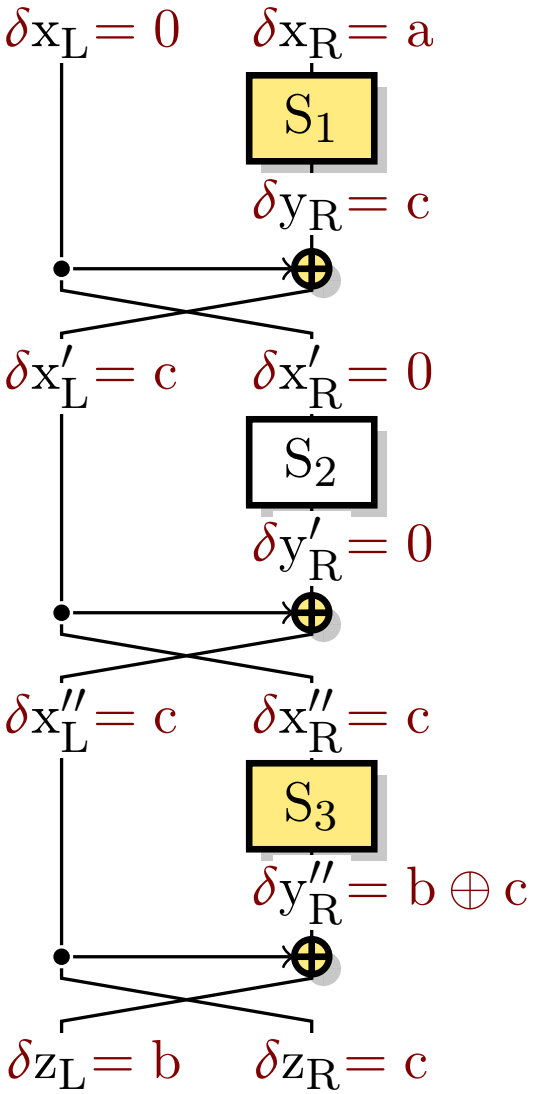
$$\delta_F(0 \,\|\, a \to b \,\|\, c) = \delta_{S_1}(a \to c) \times \delta_{S_3}(c \to b \oplus c)$$

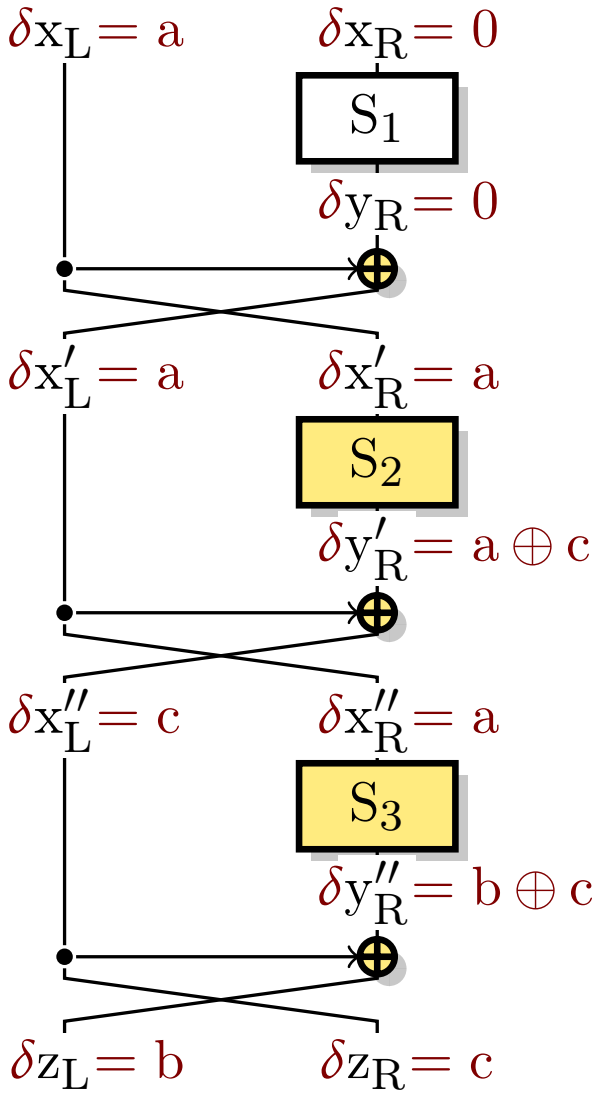
Application: if S_1 is bijective

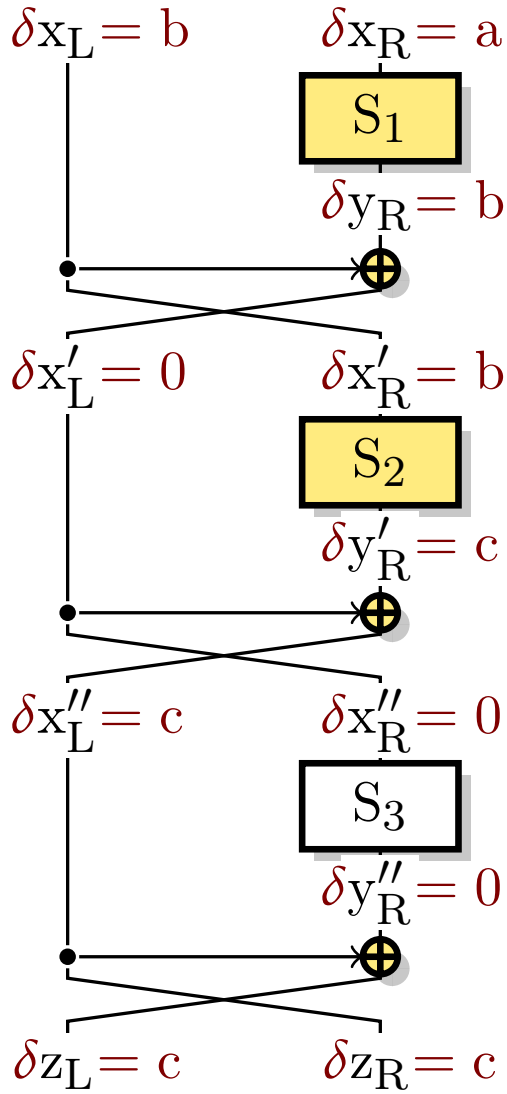
- Choose a, c such that $\delta_{S_1}(a, c) = \delta(S_1)$
- Choose b with $\delta_{S_3}(c, b \oplus c) \geq \delta_{\min}(S_3)$
- $\delta(F) \geq \delta_F(0 \,\|\, a, b \,\|\, c) \geq \delta(S_1) \times \delta_{\min}(S_3)$

- Choose b, c such that $\delta_{S_3}(c, b \oplus c) = \delta(S_3)$
- Choose a with $\delta_{S_1}(a, c) \geq \delta_{\min}(S_1^{-1})$
- $\delta(F) \geq \delta_F(0 \,\|\, a, b \,\|\, c) \geq \delta(S_3) \times \delta_{\min}(S_1^{-1})$

Sketch of Proof







Application to n = 4: Properties of 4-bit Functions

Properties of 4-bit S-Boxes

- Full classification of 4-bit permutations
  - 302 affine equivalence classes

    [De Cannière; Leander & Poschmann '07]
- Full classification of 4-bit APN functions
  - 2 extended affine equivalence classes [Brinkmann & Leander '08]
- There are 4-bit APN functions
  - $\delta(S_i) = 2$, $\delta_{min}(S_i) = 2$
- There are no 4-bit APN permutations
  - If S_i is a permutation, $\delta(S_i) \geq 4$, $\delta_{min}(S_i) \geq 2$

Refined bounds for n = 4 (MISTY and Feistel)

- If S_i are all non-bijective, then $\delta(F) \geq 32$
- If S_i bijective, $\delta(F) \geq \delta_{min}(S_j) \times \delta(S_i) \geq 8$

MISTY: Necessary Conditions for $\delta = 8$, $\mathcal{L} = 64$

Necessary Conditions for $\delta(\mathrm{F}) = 8$

- S_1 permutation with $\delta(\mathrm{S}_1) = 4$
- S_2, S_3 APN

Proof.

- Suppose $\delta(\mathrm{S}_3) \geq 4$
  - $\delta(\mathrm{S}_3) \geq 4$, therefore there exist c_1, b_1 with $\delta_{\mathrm{S}_3}(\mathrm{c}_1 \to \mathrm{b}_1) \geq 4$
  - There are two pairs $(\mathrm{x}, \mathrm{x} \oplus \mathrm{c}_1)$, $(\mathrm{y}, \mathrm{y} \oplus \mathrm{c}_1)$ in $\mathrm{D}_{\mathrm{S}_3}(\mathrm{c}_1 \to \mathrm{b}_1)$
  - With $\mathrm{c}_2 = \mathrm{x} \oplus \mathrm{y}$, $\mathrm{b}_2 = \mathrm{S}_3(\mathrm{x}) \oplus \mathrm{S}_3(\mathrm{y})$, there are two pairs (x, y), $(\mathrm{x} \oplus \mathrm{c}_1, \mathrm{y} \oplus \mathrm{c}_1)$ with $\mathrm{D}_{\mathrm{S}_3}(\mathrm{c}_2 \to \mathrm{b}_2)$
  - Similarly, there are two pairs $(\mathrm{x}, \mathrm{y} \oplus \mathrm{c}_1)$, $(\mathrm{x} \oplus \mathrm{c}_1, \mathrm{y})$ with $\mathrm{D}_{\mathrm{S}_3}(\mathrm{c}_1 \oplus \mathrm{c}_2 \to \mathrm{b}_1 \oplus \mathrm{b}_2)$
  - At least 3 lines c_i with S_3 with a value ≥ 4

MISTY: Necessary Conditions for $\delta = 8$, $\mathcal{L} = 64$

Necessary Conditions for $\delta(\mathrm{F}) = 8$

- S_1 permutation with $\delta(\mathrm{S}_1) = 4$
- S_2, S_3 APN

Proof.

- Suppose $\delta(\mathrm{S}_3) \geq 4$
  - At least 3 lines $\mathrm{c_i}$ with S_3 with a value ≥ 4
  - $\delta_{\mathrm{F}}(0||\mathrm{a} \to \mathrm{b}||\mathrm{c}) = \delta_{\mathrm{S}_1}(\mathrm{a} \to \mathrm{c}) \times \delta_{\mathrm{S}_3}(\mathrm{c} \to \mathrm{b} \oplus \mathrm{c})$
  - To get $\mathrm{c} \leftarrow \mathrm{c_i}$, we also need:
    $\mathrm{c_i}$ column of differences of S_1 with value $= 4$
  - If such a $\mathrm{c_i}$ does not exist $\Rightarrow \mathrm{L} = \{\mathrm{c}_1, \mathrm{c}_2, \mathrm{c}_3 = \mathrm{c}_1 \oplus \mathrm{c}_2\} \subseteq \mathrm{C}$,
    C = columns of S_1 without value $= 4$
  - C for the representatives of affine equivalence classes does not contain any subset stable under XOR

Constructing Strong 8-bit S-Boxes with Feistel and MISTY

Feistel

- $\delta(\mathrm{F}) \geq 8$, reached bound
  - S_1, S_3 must be APN, S_2 a permutation with $\delta(\mathrm{S}_2) = 4$
- $\mathcal{L}(\mathrm{F}) \geq 48$
  - $\mathcal{L}(\mathrm{F}) \geq 64$ if $\delta(\mathrm{F}) < 32$

MISTY

- $\delta(\mathrm{F}) \geq 8$, reached bound
  - S_2, S_3 must be APN, S_1 a permutation with $\delta(\mathrm{S}_1) = 4$
  - F is not a permutation
- $\mathcal{L}(\mathrm{F}) \geq 48$
  - $\mathcal{L}(\mathrm{F}) \geq 64$ if $\delta(\mathrm{F}) < 32$
- F permutation: $\delta(\mathrm{F}) \geq 16$, reached bound

Getting the Components

- From these results, Feistel is more adapted
- We need S_1, S_3 APN, S_2 permutation with $\delta(S_2) = 4$
  - Can we choose S_i with low implementation cost?
- Exhaustive search over small implementations until good properties are reached (Üllrich & al. 2011)
  - Search sequences of instructions for a bit-sliced implementation
  - We use equivalence classes to cut branches
  - Minimise the number of non-linear operations

Exhaustive Search Results

Permutation with $\delta = 4$

- Easy search

  Reuse results from Üllrich & al.
- 9 instructions
  - 4 non-linear
  - 4 XOR
  - 1 copy

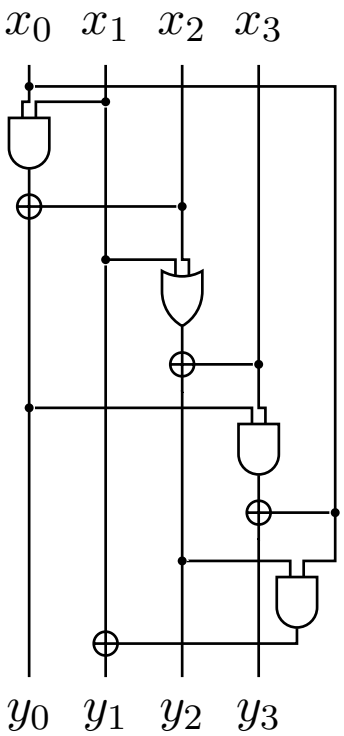
- 4 non-linear gates is optimal

APN Function

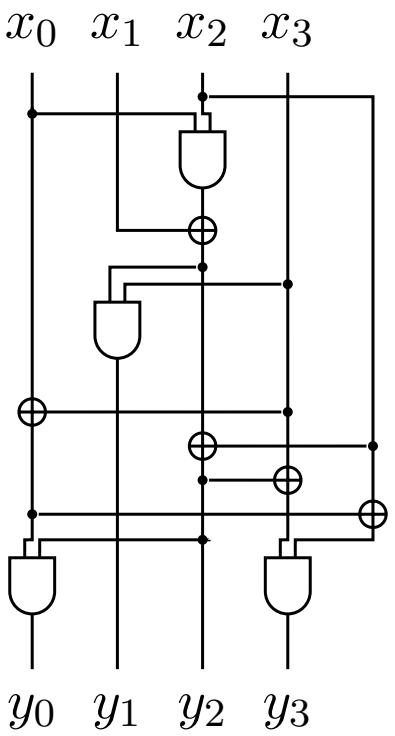
- Costly search
  - No filtering permutations
  - 6k core-hours
- 10 instructions
  - But 6 non-linear
- 11 instructions
  - 4 non-linear
  - 5 XOR
  - 2 copies

- 4 non-linear gates is optimal

Concrete Example



Permutation with $\delta = 4$ (S_2)



APN Function (S_1, S_3)

A Feistel network using these functions is an 8-bit permutation with $\delta = 8$ and $\mathcal{L} = 64$.

Results: Better than Before

		Implem.		Properties		
S-Box	Construction	$\wedge, \vee$	$\oplus$	$\mathcal{L}$	δ	Cost
AES	Inversion	32	83	32	4	1
Whirlpool	Lai-Massey	36	58	56	8	1.35
CRYPTON	3-r. Feistel	49	12	64	8	1.83
Robin	3-r. Feistel	12	24	64	16	0.56
Fantomas	3-r. MISTY (3/5 bits)	11	25	64	16	0.51
LS (unnamed)	Whirlpool-like	16	41	64	10	0.64
New	3-r. Feistel	12	26	64	8	0.45

Conclusion

1. Bounds on the security of Feistel and MISTY networks with fixed key
2. Applied to 8-bit S-Boxes
   - Necessary conditions
   - Detailed bounds for permutations
   - Feistel is better for invertible 8-bit S-Boxes
3. Concrete construction of strong light S-Boxes
   - 8-bit S-Box from 3-round Feistel
   - Better than previously used S-Boxes

Questions ?